# Constraint Logic Programming 

Sylvain Soliman<br>Sylvain.Soliman@inria.fr

Project-Team LIFEWARE

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## Part I: CLP - Introduction and Logical Background

(1) The Constraint Programming paradigm

2 Examples and Applications
(3) First Order Logic
4. Models
(5) Logical Theories

## Part II: Constraint Logic Programs

(6) Constraint Languages
(7) $\operatorname{CLP}(\mathcal{X})$
(8) $\operatorname{CLP}(\mathcal{H})$
(9) $\operatorname{CLP}(\mathcal{R}, \mathcal{F D}, \mathcal{B})$

## Part III: CLP - Operational and Fixpoint Semantics

(10) Operational Semantics
(11) Fixpoint Semantics
(12) Program Analysis

## Part IV: Logical Semantics

(13) Logical Semantics of $\operatorname{CLP}(\mathcal{X})$
(14) Automated Deduction
(15) $\operatorname{CLP}(\lambda)$
(16) Negation as Failure

## Part V: Constraint Solving

(17) Solving by Rewriting
(18) Solving by Domain Reduction

## Part VI: Practical CLP Programming

(19) CLP implementation, the WAM
(20) Optimizing CLP
(21) Symmetries
(22) Symmetry Breaking During Search
(23) Detecting Symmetries

## Part VII: More Constraint Programming

(24) Typing CLP
(25) CHR

## Part VIII: Programming Project

26 check_dice

27 dice

28 Optimizing
(29) Theory

## Part IX: Concurrent Constraint Programming

(30) Introduction
(31) Operational Semantics
(32) Examples

## Part X: CC - Denotational Semantics

(33) Deterministic Case
(34) Constraint Propagation

35 Non-deterministic Case
(36) Sequentiality

## Part XI: CC and Linear Logic

(37) CC - Logical Semantics
(38) Must Properties
(39) Program Analysis

## Intuitionistic Linear Logic

## Multiplicatives

Additives

$$
\begin{array}{lcr}
\frac{\Gamma, \boldsymbol{A} \vdash \boldsymbol{C}}{\Gamma, \boldsymbol{A} \& B \vdash C} & \frac{\Gamma, B \vdash C}{\Gamma, \boldsymbol{A} \& B \vdash C} & \frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B} \\
\frac{\Gamma, \boldsymbol{A} \vdash \boldsymbol{C}}{\Gamma, \boldsymbol{C}, \boldsymbol{B} \vdash \boldsymbol{C}} & \frac{\Gamma \vdash \boldsymbol{A}}{\Gamma, \boldsymbol{A} \oplus \boldsymbol{B} \vdash \boldsymbol{C}} & \frac{\Gamma \vdash B}{\Gamma \vdash \boldsymbol{A} \oplus B}
\end{array}
$$

Constants

$$
\begin{array}{llllll}
\frac{\Gamma \vdash A}{\Gamma, \mathbf{1} \vdash A} & \vdash \mathbf{1} & \perp \vdash & \frac{\Gamma \vdash}{\Gamma \vdash \perp} & \Gamma \vdash T & \Gamma, \mathbf{0} \vdash A
\end{array}
$$

## ILL = the Logic of CC agents

Translation:
$(A \| B)^{\dagger}=A^{\dagger} \otimes B^{\dagger} \quad(c \rightarrow A)^{\dagger}=$

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Translation:

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\begin{aligned}
& (A \| B)^{\dagger}=A^{\dagger} \otimes B^{\dagger} \quad(c \rightarrow A)^{\dagger}=c \multimap A^{\dagger} \quad \text { tell }(c)^{\dagger}=!c \\
& (A+B)^{\dagger}=
\end{aligned}
$$

## ILL = the Logic of CC agents

Translation:

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\begin{array}{ccc}
(A \| B)^{\dagger}=A^{\dagger} \otimes B^{\dagger} & (c \rightarrow A)^{\dagger}=c-A^{\dagger} & t e l(c)^{\dagger}=!c \\
(A+B)^{\dagger}=A^{\dagger} \& B^{\dagger} & (\exists x A)^{\dagger}=\exists X A^{\dagger} & p(\vec{x})^{\dagger}=p(\vec{x}) \\
& (X ; c ; \Gamma)^{\dagger}=\exists X\left(!c \otimes \Gamma^{\dagger}\right) &
\end{array}
$$

Axioms: !c $\vdash \cdot!d$ for all $c \vdash_{\mathcal{C}} d$

$$
p(\vec{x}) \vdash A^{\dagger} \text { for all } p(\vec{x})=A \in \mathcal{D}
$$

## ILL $=$ the Logic of CC agents

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Axioms: !c $\vdash$ !d for all $c \vdash_{\mathcal{C}} d \quad p(\vec{x}) \vdash A^{\dagger}$ for all $p(\vec{x})=A \in \mathcal{D}$
Soundness and Completeness
If $(c ; \Gamma) \longrightarrow C C(d ; \Delta)$ then $c^{\dagger} \otimes \Gamma^{\dagger} \vdash_{I L L(\mathcal{C}, \mathcal{D})} d^{\dagger} \otimes \Delta^{\dagger}$

If $A^{\dagger} \vdash_{I L L(C, \mathcal{D})} c$ then there exists a success store $d$ such that $($ true $; A) \longrightarrow C C(d ; \emptyset)$ and $d \vdash_{C} C$ If $A^{\dagger} \vdash_{I L L(\mathcal{C}, \mathcal{D})} \mathcal{C} \otimes \top$ then there exists an accessible store $d$ such that $($ true $; A) \longrightarrow_{C C}(d ; \Gamma)$ and $d \vdash_{C} C$

Part XII: LCC

40 LCC
(41) Examples

## $\operatorname{CC}(\mathcal{F D})$ in $\operatorname{LCC}(\mathcal{H})$

One can now easily embed in LCC our $\operatorname{CC}(\mathcal{F D})$ propagators, including

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$$
\begin{aligned}
& \text { fd }(X)=\text { tell(min }(X, \text { min_integer }) \otimes \max (X, \text { max_integer })) \\
& { }^{\prime} x \geq 1 Y+C^{\prime}(X, Y, C)=
\end{aligned}
$$

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```
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' }\textrm{x}\mp@subsup{\geq}{1}{}\textrm{Y}+\mp@subsup{\textrm{C}}{}{\prime}(\textrm{X},\textrm{Y},\textrm{C})
    min(X,MinX) \otimes min(Y,MinY) \otimes (MinX<MinY+C)
    ->(tell(min(X,MinY+C) \otimes min(Y,MinY))
    | ' X \geq 1 Y +C' (X,Y,C))
' }\textrm{x}\geq\textrm{y}+\mp@subsup{\textrm{C}}{}{\prime}(\textrm{X},\textrm{Y},\textrm{C})
```


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    ->(tell(min(X,MinY+C) \otimes min(Y,MinY))
    | ' X \geq 1 Y +C' (X,Y,C))
'x\geqy+C'(X,Y,C) = ' x \geq 1 Y + C' (X,Y,C) | ' x m 2 Y + C' (X,Y,C)
' ask (x\geqy) ->a'(X,Y,A) =
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    ->(tell(min(X,MinY+C) \otimes min(Y,MinY))
            | ' X \geq 1 Y +C' (X,Y,C))
'x\geqy+C'(X,Y,C) = 'x 
'ask(x\geqy) ->a'(X,Y,A) =
    min(X,MinX) \otimes max(Y,MaxY) \otimes (MinX\geqMaxY)
    A || tell(min(X,MinX) \otimes max(Y,MaxY))
```

Imperative variables allow a finer control, which is necessary for certain constraint solvers, e.g. the implementation of a Simplex solver in LCC [Schachter99these]

## Part XIII

## LCC Logical Semantics and more

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42 Logical Semantics
(43) Modules
(44) CHR vs. LCC

## Logical Semantics

Simple translation of LCC into ILL:

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## Logical Semantics

Simple translation of LCC into ILL:

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\begin{array}{lc}
t e l l(c)^{\dagger}=c & (A \| B)^{\dagger}=A^{\dagger} \otimes B^{\dagger} \\
\forall \vec{y}(c \rightarrow A)^{\dagger}=\forall \vec{y}\left(c \multimap A^{\dagger}\right) & p(\vec{x})^{\dagger}=p(\vec{x}) \\
(A+B)^{\dagger}=A^{\dagger} \& B^{\dagger} & (\exists x A)^{\dagger}=\exists x A^{\dagger}
\end{array}
$$

$\operatorname{ILL}(\mathcal{C}, \mathcal{D})$ denotes the deduction system obtained by adding to intuitionistic linear logic the axioms:

- $c \vdash d$ for every $c \Vdash_{\mathcal{C}} d$ in $\Vdash_{\mathcal{C}}$,
- $p(\vec{X}) \vdash A^{\dagger}$ for every declaration $p(\vec{x})=A$ in $\mathcal{D}$.

Same soundness/completeness results as for CC.

## Phase Semantics

A phase space $\mathbf{P}=\langle P, \times, 1, \mathcal{F}\rangle$ is a structure such that:
(1) $\langle P, \times, 1\rangle$ is a commutative monoid.
(2) the set of facts $\mathcal{F}$ is a subset of $\mathcal{P}(P)$ such that: $\mathcal{F}$ is closed by arbitrary intersection, and for all $A \subset P$, for all $F \in \mathcal{F}$, $A \multimap F \triangleq\{x \in P: \forall a \in A, a \times x \in F\}$ is a fact.
We define the following operations:

$$
A \& B \triangleq A \cap B
$$

$$
\begin{aligned}
A \otimes B \triangleq \bigcap\{F \in \mathcal{F}: A \times B \subset F\} & A \oplus B \triangleq \bigcap\{F \in \mathcal{F}: A \cup B \subset F\} \\
\exists x A \triangleq \bigcap\left\{F \in \mathcal{F}:\left(\bigcup_{x} A\right) \subset F\right\} & \forall x A \triangleq \bigcap\left\{F \in \mathcal{F}:\left(\bigcap_{x} A\right) \subset F\right\}
\end{aligned}
$$

We'll note $T \triangleq P, \mathbf{o} \triangleq \bigcap\{F \in \mathcal{F}\}$ and $\mathbf{1} \triangleq \bigcap\{F \in \mathcal{F} \mid 1 \in F\}$.

## Interpretation

Let $\eta$ be a valuation assigning a fact to each atomic formula such that: $\eta(T)=T, \eta(\mathbf{1})=\mathbf{1}$ and $\eta(\mathbf{o})=\mathbf{o}$.
We can now define inductively the interpretation of a sequent:

$$
\begin{aligned}
\eta(\Gamma \vdash \boldsymbol{A}) & =\eta(\Gamma) \multimap \eta(\boldsymbol{A}) & & \eta(\Gamma)=\mathbf{1} \text { if } \Gamma \text { is empty } \\
\eta(\Gamma, \Delta) & =\eta(\Gamma) \otimes \eta(\Delta) & & \eta(\boldsymbol{A} \otimes \boldsymbol{B})=\eta(\boldsymbol{A}) \otimes \eta(\boldsymbol{B}) \\
\eta(\boldsymbol{A} \& \boldsymbol{B}) & =\eta(\boldsymbol{A}) \& \eta(\boldsymbol{B}) & & \eta(\boldsymbol{A} \multimap \boldsymbol{B})=\eta(\boldsymbol{A}) \multimap \eta(\boldsymbol{B})
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We then define the notion of validity as follows:
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\end{aligned}
$$

We then define the notion of validity as follows:
$\mathbf{P}, \eta \vDash(\Gamma \vdash \boldsymbol{A})$ iff $1 \in \eta(\Gamma \vdash \boldsymbol{A})$, thus $\eta(\Gamma) \subset \eta(\boldsymbol{A})$.
Soundness:

$$
\Gamma \vdash_{I L L} A \text { implies } \forall \mathbf{P}, \forall \eta, \mathbf{P}, \eta \vDash(\Gamma \vdash A) .
$$

(syntactic proof for completeness)

## Phase Counter-Models

We impose to every valuation $\eta$ to satisfy the non-logical axioms of ILL $\mathcal{C}_{\mathcal{C}, \mathcal{D}}$ :

- $\eta(c) \subset \eta(d)$ for every $c \Vdash_{\mathcal{C}} d$ in $\Vdash_{\mathcal{C}}$,
- $\eta(\boldsymbol{p}) \subset \eta\left(\boldsymbol{A}^{\dagger}\right)$ for every declaration $p=\boldsymbol{A}$ in $\mathcal{D}$.

The contrapositive of the two soundness theorems becomes:

## Theorem 1

to prove a safety property of the form

$$
(X ; c ; A) \nrightarrow(Y ; d ; B)
$$

It is enough to show

$$
\exists \mathbf{P}, \exists \eta, \exists a \in \eta\left((X ; c ; A)^{\dagger}\right) \text { such that } a \notin \eta\left((Y ; \boldsymbol{d} ; B)^{\dagger}\right) \text {. }
$$

## Producer Consumer Protocol in LCC

$\mathrm{P}=\operatorname{dem} \rightarrow($ pro $\| \mathrm{P})$
$\mathrm{C}=$ pro $\rightarrow(\operatorname{dem} \| \mathrm{C})$
init $=\operatorname{dem}^{n}\left\|\mathrm{P}^{m}\right\| \mathrm{C}^{k}$

Deadlock-freeness: init $\longrightarrow \operatorname{dem}^{n^{\prime}}\left\|\mathrm{P}^{m^{\prime}}\right\| \mathrm{C}^{k^{\prime}} \|$ pro ${ }^{\prime \prime}$, with either $n^{\prime}=l^{\prime}=0$ or $m^{\prime}=0$ or $k^{\prime}=0$

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Let us consider the structure $(\mathbb{N}, \times, 1, \mathcal{P}(\mathbb{N}))$, it is obviously a phase space.

## Producer Consumer Protocol in LCC

```
P = dem }->\mathrm{ (pro | P)
C = pro }->\mathrm{ (dem | C)
init = dem}\mp@subsup{}{}{n}|\mp@subsup{P}{}{m}|\mp@subsup{C}{}{k
```

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Let us consider the structure $(\mathbb{N}, \times, 1, \mathcal{P}(\mathbb{N}))$, it is obviously a phase space.

Let us define the following valuation:

$$
\begin{gathered}
\eta(\mathrm{P})=\{2\} \quad \eta(\mathrm{c})=\{3\} \quad \eta(\mathrm{dem})=\{5\} \quad \eta(\text { pro })=\{5\} \\
\eta(\text { init })=\left\{2^{m} \cdot 3^{k} \cdot 5^{n}\right\}
\end{gathered}
$$

## Proof

We have to check the correctness of $\eta$ :
$\forall p_{1} \in \eta(\mathrm{P}), \exists p_{2} \in \eta(\mathrm{P})$, dem $\cdot p_{1}=$ pro $\cdot p_{2}$, hence $\eta(\mathrm{P}) \subset \eta$ (body of P ).
The same for C , and $\eta$ (init) $=\eta$ (body of init).

Instead of exhibiting a counter-example, we prove $A b$ absurdum the impossibility of the inclusion

$$
\eta(\text { init }) \subset \eta\left(\operatorname{dem}^{n^{\prime}}\left\|\mathrm{P}^{m^{\prime}}\right\| \mathrm{C}^{k^{\prime}} \| \mathrm{pro}^{\prime \prime}\right)
$$

## Proof (cont.)

Suppose $\eta($ init $) \subset\left\{5^{n^{\prime}} \cdot 2^{m^{\prime}} \cdot 3^{k^{\prime}} \cdot 5^{\prime \prime}\right\}$

Since $\eta$ (init) $=\left\{2^{m} \cdot 3^{k} \cdot 5^{n}\right\}$
anything else than: $n^{\prime}+l^{\prime}=n$ and $m^{\prime}=m$ and $k^{\prime}=k$ is impossible
now note that if there is a deadlock we have:
$n^{\prime}+l^{\prime}=0 \neq n$, or $m^{\prime}=0 \neq m$, or $k^{\prime}=0 \neq k$
$\eta$ (init) is thus not a subset of the interpretation of any deadlock and thus init does not reduce into it,

## Automatization

The search for a phase space can be automatized, if one accepts some restrictions:

- always use the structure $(\mathbb{N}, \times, 1, \mathcal{P}(\mathbb{N}))$;


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The search for a phase space can be automatized, if one accepts some restrictions:

- always use the structure $(\mathbb{N}, \times, 1, \mathcal{P}(\mathbb{N})$ ); [be careful that integers are invertible]
- always look for simple (singleton/doubleton/finite) interpretations.
[might lead to confusions]


## Declarations as agents

Processes $\quad P::=\mathcal{D} . A$
Declarations $\mathcal{D}::=p(\vec{x})=A, \mathcal{D} \mid \epsilon$
Agents $\quad A::=\operatorname{tell}(c)|\forall \vec{x}(c \rightarrow A)| A \| A|\exists x A| A+A \mid p(\vec{x})$
becomes

Processes $\quad A::=\operatorname{tell}(c)|\forall \vec{x}(c \rightarrow A)| A \| A|\exists x A| \forall \vec{x}(c \Rightarrow A)$

Operational semantics of persistent asks is the same as that of asks except that the agent is not consumed.

Local choice

## Declarations as agents

$$
\begin{array}{ll}
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\text { Agents } & A::=\operatorname{tell}(c)|\forall \vec{x}(c \rightarrow A)| A \| A|\exists x A| A+A \mid p(\vec{x})
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Operational semantics of persistent asks is the same as that of asks except that the agent is not consumed.

Local choice can be encoded through asks:
$A+B=\exists x($ tell $(\operatorname{choice}(x))\|\operatorname{choice}(x) \rightarrow A\| \operatorname{choice}(x) \rightarrow B)$

## Closures as persistent asks

A closure is simply some code with an environment. The persistent ask and the hiding mechanism provide just that.

```
forall iterator
forall([]) => tell(true) |
\forall H , T \text { forall([H\|T]) = tell(apply (H)) \| tell(forall(T)) \|}
\forallx(apply(x) = Body(x))
```

This idea provides a simple encoding of declarations, but also of multi-headed rules as agents

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\forallx(apply(x) = Body(x))
```

This idea provides a simple encoding of declarations, but also of multi-headed rules as agents (CHR).

Observables definition leads to separating the constraints in order to project "process calls" and distinguish declarations from usual suspensions.

## Modules as closures

The closure mechanism provides a natural encoding of modules as first class citizens of LCC by simply considering the first argument of predicates as "module name".

Can be used for CLP too (see [HFO6iclp]) with better properties w.r.t. meta-predicates than usual module systems (e.g. SICStus)

The scope of module declarations is given by the scope of the corresponding variable.

There are two problems however with this module system :

- unification


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The scope of module declarations is given by the scope of the corresponding variable.

There are two problems however with this module system :

- unification $\Rightarrow$ union of clauses;
- module name capture with $\forall$


## Two sides of the same coin

Protect the implementation from the outside context.

Do not allow external calls to a predicate that is not exported (private).

Protect the outside context from being accessed by the implementation.

Do not allow unrestricted access to the calling context (variables) from inside the implementation.

## Code protection

To enforce code protection a simple technique is to restrict the syntax and the constraint system:

- No universal quantification on module variables (MLCC)
- No constraints making "all variables equal"

If we enforce the second one by imposing that $\{x, y\} \subset f v(c)$ whenever $c \vdash_{\mathcal{C}} x=y \otimes T$, we get :

```
Theorem 2 (Code protection [HFS07fsttcs])
Let \(A\) and \(B\) be two MLCC agents. If \(A\) has no inner module and \(y\) is used in \(A\) and \(B\) only in modular tells of the form \(y: I\) with \(y \notin f v(I)\), then \(A\) is protected in \(\exists y(y\{A\} \| B)\).
```


## SICStus/SWI modules do not offer any code protection

:- module(library, [mycall/1]).

```
:- module(using, [test/0])
:- use_module(library).
p :- write('using:p/0ч⿺').
q :- write('using:q/0ч⿺').
test :-
    library:p,
    mycall(q).
```

Unlimited qualification.
The meta-predicate declaration even allows for dynamic qualification.

```
| ? using:test.
library:p/0 using:p/0 using:q/0
yes
```


## ECLiPSe modules do not either

:- module(library, [mycall/1]).

```
:- module(using, [test/0]).
:- use_module(library).
p :- write('using:p/0').
q :- write('using:q/0').
test :-
    call(p)@library,
    mycall(q).
```

Only exported predicates accessible through qualification, but unlimited call@ construct.
The tool declaration allows for dynamic qualification.

```
| ? using:test.
library:p/0 using:p/0 using:q/0
yes
```


## EMoP modules

EMoP is the implementation by T. Martinez of [HFS07fsttcs] http://lifeware.inria.fr/~tmartine/emop/
module 'data.ref.non_backtrackable' \{
new(Initial, Ref) :-
'kernel':ref_non_backtrackable_new(Initial, X),
module Ref [Ref, X] \{
get (V) :-
set (V) :-
\}.
\}

CLP with modules (and closures) as first-class objects, including unification, passing around, environment, etc. Bonus: functional syntax, modular and redefinable, fully bootstrapped, compiled to native, ...

## CSR $\Leftrightarrow$ flat-LCC

CSR is the fragment of CHR with only simplification rules:

$$
\frac{(H \Leftrightarrow C \mid B)[x / y] \in P \quad \mathcal{T} \vDash G_{\text {builtin }} \supset \exists x\left(H=H^{\prime} \wedge C\right)}{H^{\prime} \wedge G \longrightarrow G \wedge H=H^{\prime} \wedge B}
$$

Equivalent to full CHR as far as original operational semantics (and linear logic semantics) are concerned.
[Martinez09chr] shows that CSR can be encoded in LCC:

$$
(H \Leftrightarrow C \mid B)^{\dagger}=\forall \vec{y}\left(C^{\dagger} \otimes H^{\dagger} \Rightarrow \exists \vec{x} \cdot B^{\dagger}\right)
$$

where $\vec{x}=f v(B) \backslash f v\left(H^{\prime}, C\right)$ and $\vec{y}=f v\left(H^{\prime}, C\right)$
The encoding is reciprocal for flat-LCC, i.e., LCC with all asks at top-level.
$\mathrm{LCC} \Leftrightarrow$ flat-LCC
Actually LCC itself can be encoded in flat-LCC:

- label each (persistent or not) ask with a new token depending on the free variables it depends on
- move all asks to top-level, adding to their guard the corresponding label
- add tells after each ask for all asks under it

Both bisimilarity and semantics preservation hold [Martinez09chr] (Coq proof)

PS: Marelle - Logic Programming for devops Made HN front page in September 2013.
http://quietlyamused.org/blog/2013/11/09/
marelle-for-devops/
"At 99designs [...] machines should be disposable. This requires the entire setup of a new machine to be automated.

At first I amassed shell scripts of complicated install routines, and whilst these worked they weren't that composable, say when you wanted multiple services on the same machine. Then from Babushka we learned a better way: test if something you need's there, install it if it's not, then test again to see if you succeeded. This is not hugely different from using make, just more flexible and more fault-tolerant.

Still, Babushka made me uneasy: all this ceremony and complex templating, just to describe a few facts and simple rules?" - Lars Yencken

