About Feature Trees

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Internal Seminar – 17 March 2008
1. \(\psi\)-terms

2. Ordering and Unification

3. Attribute-Value Logic
Eventually cyclic terms, with no constrained arity.

About $\psi$-terms

\begin{center}
\begin{tikzpicture}


\node[shape=circle,draw=black,minimum size=0.5cm] (person) at (0,0) {person};
\node[shape=circle,draw=black,minimum size=0.5cm] (string1) at (3,0) {string};
\node[shape=circle,draw=black,minimum size=0.5cm] (string2) at (6,0) {string};
\node[shape=circle,draw=black,minimum size=0.5cm] (date) at (9,1.5) {date};
\node[shape=circle,draw=black,minimum size=0.5cm] (unicode) at (9,-1.5) {unicode};

\draw[->] (person) -- (string1);
\draw[->] (person) -- (string2);
\draw[->] (string1) -- (date);
\draw[->] (string2) -- (date);
\draw[->] (date) -- (unicode);
\draw[->] (person) .. controls (1.5,1.5) .. (string1);
\draw[->] (person) .. controls (1.5,-1.5) .. (string2);
\draw[->] (string1) .. controls (5.5,0.5) .. (string2);
\draw[->] (string2) .. controls (5.5,-0.5) .. (string1);

\node[below] at (1.5,1.5) {FIRSTNAME};
\node[below] at (1.5,-1.5) {LASTNAME};
\node[below] at (3,0) {ENCODING};
\node[below] at (6,0) {ENCODING};
\node[below] at (9,1.5) {ENCODING};
\node[below] at (0,0) {MOTHER};
\node[below] at (3,0) {DATE};
\node[below] at (6,0) {DATE};
\node[below] at (9,1.5) {DATE};
\node[below] at (9,-1.5) {DATE};
\node[above] at (0,0) {FATHER};
\end{tikzpicture}
\end{center}
AVA Notation

1 person

Birthday : [date]

LastName :

ENCODING : [2 unicode]

FirstName :

string

ENCODING : [2]

Father :

MOTHER :

1

1

2

unicode

string

About Feature Trees
Definition

Let \((\text{TYPE}, \sqsubseteq)\) be a BCPO and \text{FEAT} be a finite set of features. A \(\psi\)-term is a tuple \((Q, \overline{q}, \theta, \delta)\) such that:

- \(Q\) is a finite set of states \emph{rooted} by \(\overline{q}\).
- \(\overline{q}\) is the root of the term.
- \(\theta : Q \rightarrow \text{TYPE}\) is the \emph{type} function.
- \(\delta : Q \times \text{FEAT} \rightarrow Q\) is the \emph{value} function.

\(\theta\) is a total function. \(\delta\) is a partial function.
Let $\delta^*: Q \times \text{FEAT}^* \to Q$ the partial function extending $\delta$ to paths:

$$
\delta^*(q, \varepsilon) = q, \delta^*(q, f\pi) = \delta^*(\delta(f, \pi), \pi).
$$

**Definition (Rooted)**

$Q$ is *rooted* by $\overline{q}$ if

$$
Q = \{ \delta^*(\overline{q}, \pi) \mid \pi \in \text{FEAT}^* \text{ such that } \delta^*(\overline{q}, \pi) \text{ is defined} \}.
$$
Subsumption

Definition

\[ F = (Q, \bar{q}, \delta, \theta) \] subsumes \( F' = (Q', \bar{q}', \delta', \theta') \), \( F \sqsubseteq F' \), if there is a morphism \( h : Q \rightarrow Q' \):

1. \( h(\bar{q}) = \bar{q}' \).
2. \( \theta(q) \sqsubseteq \theta'(h(q)) \) for every \( q \in Q \).
3. \( h(\delta(f, q)) = \delta'(f, h(q)) \) for every \( q \in Q, f \in \text{FEAT} \) such that \( \delta(f, q) \) is defined.

We write \( F \sim F' \) if \( F \sqsubseteq F' \) and \( F' \sqsubseteq F \).
We have $F \sqsubseteq F'$ but not $F \sim F'$!
**Definition**

Let be $F_1 \sim (Q_1, \overline{q_1}, \delta_1, \theta_1)$ and $F_2 \sim (Q_2, \overline{q_2'}, \delta_2', \theta_2')$ such that $F_1 \cap F_2 = \emptyset$.

$$F_1 \sqcup F_2 = ((Q_1 \cup Q_2)/\asymp, \overline{q_1}, \theta^{\asymp}, \delta^{\asymp})$$

where

- $\asymp$ is the least equivalence relation such that:
  1. $\overline{q_1} \asymp \overline{q_2}$
  2. $\delta_1(f, q_1) \asymp \delta(f, q_2)$ if both are defined and $q_1 \asymp q_2$

- $\theta^{\asymp}([q]^{\asymp}) = \sqcup(\{ \theta_1(q_1) \mid q \asymp q_1 \text{ and } \theta_1(q_1) \text{ is defined} \}) \cup \{ \theta_2(q_2) \mid q \asymp q_2 \text{ and } \theta_2(q_2) \text{ is defined} \}$

- $\delta^{\asymp}([q_1]^{\asymp}) = [\delta_1(f, q_1)]^{\asymp}$ if $\delta_1(f, q_1)$ is defined;
- $\delta^{\asymp}([q_2]^{\asymp}) = [\delta_2(f, q_2)]^{\asymp}$ if $\delta_2(f, q_2)$ is defined.
Definition (Descriptions)

The least set $\text{DESC}$ such that:

- $\sigma \in \text{DESC}$ if $\sigma \in \text{TYPE}$
- $\pi : \phi \in \text{DESC}$ if $\pi \in \text{FEAT}^*$, $\phi \in \text{DESC}$
- $\pi_1 = \pi_2 \in \text{DESC}$ if $\pi_1, \pi_2 \in \text{FEAT}^*$
- $\phi \land \psi \in \text{DESC}$ if $\phi, \psi \in \text{DESC}$
Definition (Semantics)

Let $F$ be a $\psi$-term.

- $F \models \sigma$ if $\sigma \in \text{TYPE}$ and $\sigma \sqsubseteq \theta(\overline{q})$
- $F \models \pi : \psi$ if the substructure $F'$ of $F$ following the path $\pi$ is such that $F' \models \psi$
- $F \models \pi_1 = \pi_2$ if $\delta(\pi_1, (q)) = \delta(\pi_2, (q))$
- $F \models \phi \land \psi$ if $F \models \phi$ and $F \models \psi$
Most General Satisfier

**Theorem**

There is a partial function $MGSat : \text{DESC} \rightarrow \psi\text{-Terms}$ such that:

$$F \models \psi$$

if and only if $MGSat(\psi) \sqsubseteq F$

**Theorem**

$MGSat(\text{DESC}) = \psi\text{-Terms}$
Pros:
- Types hierarchies to describe the unification process of values
- Correspondance between Subsumption and Unification
- Correspondance between Subsumption and Attribute-Value Logic
- High-Performance Unification Algorithms

Cons:
- Not constraint-based: lazy evaluation of projection
- Features are atomic