About Feature Trees

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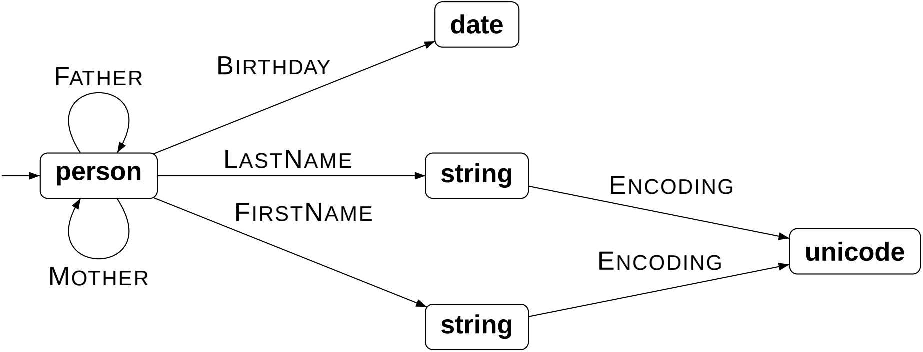
About ψ-terms

Eventually cyclic terms, with no constrained arity.



AVM Notation

$$\begin{bmatrix} \boxed{1}\ \textbf{person} \\ \textsc{Birthday} : \begin{bmatrix}\textbf{date}\end{bmatrix} \\ \textsc{LastName} : \begin{bmatrix} \textbf{string} \\ \textsc{Encoding} : \begin{bmatrix} \boxed{2}\ \textbf{unicode} \end{bmatrix} \end{bmatrix} \\ \textsc{FirstName} : \begin{bmatrix} \textbf{string} \\ \textsc{Encoding} : \begin{bmatrix} \boxed{2} \end{bmatrix} \end{bmatrix} \\ \textsc{Father} : \begin{bmatrix} \boxed{1} \end{bmatrix} \\ \textsc{Mother} : \begin{bmatrix} \boxed{1} \end{bmatrix} \end{bmatrix}$$

Formal Definition

Definition

Let $(\textsc{Type}, \sqsubseteq)$ be a BCPO and $\textsc{Feat}$ be a finite set of features. A ψ-term is a tuple $(Q, \overline{q}, \theta, \delta)$ such that :

- Q is a finite set of states *rooted* by $\overline{q}$.
- $\overline{q}$ is the root of the term.
- $\theta : Q \longrightarrow \textsc{Type}$ is the *type* function.
- $\delta : Q \times \textsc{Feat} \longrightarrow Q$ is the *value* function.

θ is a total function. δ is a partial function.

Feature Structures are connex

Let $\delta^* : Q \times \textsc{Feat}^* \longrightarrow Q$ the partial function extending δ to paths :
$\delta^*(q,\varepsilon) = q, \delta^*(q,f\pi) = \delta^*(\delta(f,\pi),\pi)$.

Definition (Rooted)

Q is *rooted* by $\overline{q}$ if

$$Q = \left\{ \delta^*(\overline{q},\pi) \;\middle|\; \pi \in \textsc{Feat}^* \text{ such that } \delta^*(\overline{q},\pi) \text{ is defined} \right\}$$

Subsumption

Definition

$F = (Q, \overline{q}, \delta, \theta)$ subsumes $F' = (Q', \overline{q}', \delta', \theta')$, $F \sqsubseteq F'$, if there is a morphism $h : Q \rightarrow Q'$:

- $h(\overline{q}) = \overline{q}'$.
- $\theta(q) \sqsubseteq \theta'(h(q))$ for every $q \in Q$.
- $h(\delta(f, q)) = \delta'(f, h(q))$ for every $q \in Q$, $f \in \textsc{Feat}$ such that $\delta(f, q)$ is defined.

We write $F \sim F'$ if $F \sqsubseteq F'$ and $F' \sqsubseteq F$.

Identity

$$F = \begin{bmatrix} \boxed{1}\ \mathbf{a} \\ \textsc{Arg} : \left[\boxed{1}\right] \end{bmatrix}$$

$$F' = \begin{bmatrix} \mathbf{a} \\ \textsc{Arg} : \begin{bmatrix} \boxed{1}\ \mathbf{a} \\ \textsc{Arg} : \left[\boxed{1}\right] \end{bmatrix} \end{bmatrix}$$

We have $F \sqsubseteq F'$ but not $F \sim F'$!

ψ-term Unification

Definition

Let be $F_1 \sim (Q_1, \overline{q_1}, \delta_1, \theta_1)$ and $F_2 \sim (Q_2, \overline{q_2}', \delta_2', \theta_2')$ such that $F_1 \cap F_2 = \emptyset$.

$$F_1 \sqcup F_2 = ((Q_1 \cup Q_2)/\bowtie, [\overline{q_1}]_{\bowtie}, \theta^{\bowtie}, \delta^{\bowtie})$$

where

- $\bowtie$ is the least equivalence relation such that :
  1. $\overline{q_1} \bowtie \overline{q_2}$
  2. $\delta_1(f, q_1) \bowtie \delta(f, q_2)$ if both are defined and $q_1 \bowtie q_2$
- $\theta^{\bowtie}([q]_{\bowtie}) = \sqcup(\{\theta_1(q_1) \mid q \bowtie q_1 \text{ and } \theta_1(q_1) \text{ is defined}\} \cup \{\theta_2(q_2) \mid q \bowtie q_2 \text{ and } \theta_2(q_2) \text{ is defined}\})$
- $\delta^{\bowtie}([q_1]_{\bowtie}) = [\delta_1(f, q_1)]_{\bowtie}$ if $\delta_1(f, q_1)$ is defined ; $\delta^{\bowtie}([q_2]_{\bowtie}) = [\delta_2(f, q_2)]_{\bowtie}$ if $\delta_2(f, q_2)$ is defined.

Description Language

Definition (Descriptions)

The least set DESC such that:

- $\sigma \in$ DESC if $\sigma \in$ TYPE
- $\pi : \phi \in$ DESC if $\pi \in$ FEAT*, $\phi \in$ DESC
- $\pi_1 = \pi_2 \in$ DESC if $\pi_1, \pi_2 \in$ FEAT*
- $\phi \wedge \psi \in$ DESC if $\phi, \psi \in$ DESC

Description Language Semantics

Definition (Semantics)

Let F be a ψ-term.

- $F \models \sigma$ if $\sigma \in \textsc{Type}$ and $\sigma \sqsubseteq \theta(\overline{q})$
- $F \models \pi : \psi$ if the substructure F' of F following the path π is such that $F' \models \psi$
- $F \models \pi_1 = \pi_2$ if $\delta(\pi_1, \overline{(}q)) = \delta(\pi_2, \overline{(}q))$
- $F \models \phi \wedge \psi$ if $F \models \phi$ and $F \models \psi$

Most General Satisfier

Theorem

There is a partial function MGSat : DESC $\to$ ψ*-Terms such that:*

$$F \models \psi$$

if and only if $MGSat(\psi) \sqsubset F$

Theorem

$MGSat(\text{DESC}) = \psi$*-Terms*

Conclusion

Pros:

- Types hierarchies to describe the unification process of values
- Correspondance between Subsumption and Unification
- Correspondance between Subsumption and Attribute-Value Logic
- High-Performance Unification Algorithms

Cons:

- Not constraint-based: lazy evaluation of projection
- Features are atomic