About Feature Trees

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1. $\psi$-terms

2. Ordering and Unification

3. Attribute-Value Logic
Eventually cyclic terms, with no constrained arity.
AVM Notation

```
[1] person
  BIRTHDAY: [date]
  LASTNAME:
    string
    ENCODING: [2 unicode]
  FIRSTNAME:
    string
    ENCODING: [2]
  FATHER: [1]
  MOTHER: [1]
```
Definition

Let \((\text{TYPE}, \sqsubseteq)\) be a BCPO and \(\text{FEAT}\) be a finite set of features. A \(\psi\)-term is a tuple \((Q,q,\theta,\delta)\) such that:

- \(Q\) is a finite set of states *rooted* by \(\bar{q}\).
- \(\bar{q}\) is the root of the term.
- \(\theta : Q \rightarrow \text{TYPE}\) is the *type* function.
- \(\delta : Q \times \text{FEAT} \rightarrow Q\) is the *value* function.

\(\theta\) is a total function. \(\delta\) is a partial function.

About Feature Trees
Feature Structures are connex

Let $\delta^* : Q \times \text{FEAT}^* \rightarrow Q$ the partial function extending $\delta$ to paths:

$\delta^* (q, \varepsilon) = q$, $\delta^* (q, f \pi) = \delta^* (\delta (f, \pi), \pi)$.

**Definition (Rooted)**

$Q$ is *rooted* by $\overline{q}$ if

$$Q = \{ \delta^* (\overline{q}, \pi) \mid \pi \in \text{FEAT}^* \text{ such that } \delta^* (\overline{q}, \pi) \text{ is defined} \}$$
We have $F \sqsubseteq F'$ but not $F \sim F'$!
\( \psi \text{-term Unification} \)

**Definition**

Let be \( F_1 \sim (Q_1, \overline{q}_1, \delta_1, \theta_1) \) and \( F_2 \sim (Q_2, \overline{q}'_2, \delta'_2, \theta'_2) \) such that \( F_1 \cap F_2 = \emptyset \).

\[
F_1 \sqcup F_2 = ((Q_1 \cup Q_2) / \bowtie, [\overline{q}_1], \theta^\bowtie, \delta^\bowtie)
\]

where

- \( \bowtie \) is the least equivalence relation such that:
  1. \( \overline{q}_1 \bowtie \overline{q}_2 \)
  2. \( \delta_1(f, q_1) \bowtie \delta(f, q_2) \) if both are defined and \( q_1 \bowtie q_2 \)

- \( \theta^\bowtie([q]_{\bowtie}) = \sqcup(\{\theta_1(q_1) \mid q \bowtie q_1 \text{ and } \theta_1(q_1) \text{ is defined}\} \cup \{\theta_2(q_2) \mid q \bowtie q_2 \text{ and } \theta_2(q_2) \text{ is defined}\}) \)

- \( \delta^\bowtie([q_1]_{\bowtie}) = [\delta_1(f, q_1)]_{\bowtie} \) if \( \delta_1(f, q_1) \) is defined;
  \( \delta^\bowtie([q_2]_{\bowtie}) = [\delta_2(f, q_2)]_{\bowtie} \) if \( \delta_2(f, q_2) \) is defined.
Description Language

Definition (Descriptions)

The least set $\text{DESC}$ such that:

- $\sigma \in \text{DESC}$ if $\sigma \in \text{TYPE}$
- $\pi : \phi \in \text{DESC}$ if $\pi \in \text{FEAT}^*$, $\phi \in \text{DESC}$
- $\pi_1 = \pi_2 \in \text{DESC}$ if $\pi_1, \pi_2 \in \text{FEAT}^*$
- $\phi \land \psi \in \text{DESC}$ if $\phi, \psi \in \text{DESC}$
Definition (Semantics)

Let $F$ be a $\psi$-term.

- $F \models \sigma$ if $\sigma \in \text{TYPE}$ and $\sigma \sqsubseteq \theta(\overline{q})$
- $F \models \pi : \psi$ if the substructure $F'$ of $F$ following the path $\pi$ is such that $F' \models \psi$
- $F \models \pi_1 = \pi_2$ if $\delta(\pi_1, \overline{q}) = \delta(\pi_2, \overline{q})$
- $F \models \phi \land \psi$ if $F \models \phi$ and $F \models \psi$
Most General Satisfier

**Theorem**

There is a partial function \( \text{MGSat} : \text{DESC} \rightarrow \psi\text{-Terms} \) such that:

\[
\text{if and only if } \text{MGSat}(\psi) \sqsubseteq F
\]

**Theorem**

\( \text{MGSat(D}_{\text{ESC}}) = \psi\text{-Terms} \)
Conclusion

Pros:

- Types hierarchies to describe the unification process of values
- Correspondance between Subsumption and Unification
- Correspondance between Subsumption and Attribute-Value Logic
- High-Performance Unification Algorithms

Cons:

- Not constraint-based: lazy evaluation of projection
- Features are atomic