The Cream Type System

Thierry Martinez
Acknowledgments to Julien Martin

Contraintes Project–Team
INRIA Paris–Rocquencourt Research Centre

Wednesday 18th November 2009

1 Introduction
2 The Cream Type System
3 A Hindley-Milner based type inference algorithm
4 Conclusion
The Cream Modelling Language

- A rule-based language for modelling satisfaction and optimisation problems. [RAC’09]
- The language enjoys directives for the declarative specification of search-heuristics. [CPAIOR’09]
- Very few data structures: fd variables, integer constants, lists and records.
- Aggregators over lists. No recursion.


The Cream Modelling Language

**Object definitions**

```
queen(I) = { row = I, column = _ }.
board(N) = map(I, [1 .. N], queen(I)).
```

**Rule declarations**

```
no_attack(Q0, Q1) -->
    Q0:column # Q1:column
    and Q0:row - Q0:column # Q0:row - Q1:column
    and Q0:row + Q0:column # Q0:row + Q1:column.

no_attack(L) -->
    forall(Q0 in L,
        forall(Q1 in L,
            Q0:row < Q1:row => no_attack(Q0, Q1))).
```

**Query**

```
? let (N = 10,
    Board = board(N),
    domain(Board, 1, N) and no_attack(Board)
    and labelling(Board)).
```
Cream Type Constructors

- **int** the type of integer constants and finite domain variables
  - 1, 10, I, N :: int

- **bool** the type of constraints and rules (truth values)
  - Q0:column # Q1:column :: bool
  - Q0:row # Q1:row => no_attack(Q0, Q1) :: bool
  - 1 :: bool

- **[ τ ]** the type of lists with elements of type τ (homogeneous lists)
  - [1 .. N] :: [int]

- **{ f₁: τ₁, ..., fₙ: τₙ}** the type of records with a field f₁ carrying a value of type τ₁, ..., a field fₙ carrying a value of type τₙ
  - queen(I) = { row = I, column = _ }.
  - queen(α) :: { row: α, column: β}
  - board(N) = map(I, [1 .. N], queen(I))
  - board(int) :: [{ row: int, column: α}]
What Kind Of Guarantees Could Be Expected?

- Type consistency in a call:
  \[\text{board}(\text{int}) :: \{\text{row} : \text{int}, \text{column} : \alpha\}\]
  \[\text{board}([1 .. 10])\] should fail to type.

- Projection validity: \(\text{queen}(\alpha) :: \{\text{row} : \alpha, \text{column} : \beta\}\)
  \(\text{queen}(N) : \text{column}\) should fail to type.

- Object construction validity:
  \(\text{no\_attack}([\{\text{row} : \text{int}, \text{column} : \text{int}\}]) :: \text{bool}\)
  \(\text{no\_attack}([\{\text{line} = 4, \text{column} = 2\}])\) should fail.

But: a well-typed Cream program can go wrong (with respect to the rewriting system \(\rightarrow\))
\(\text{nth}(1, [])\) is well-typed but \(\text{nth}(1, []) \not\rightarrow\).
What is a type judgement?

The type system associates a type to every (well-typed) expression (e.g. \(1 + 1 :: \text{int}\)).

Expressions may depend on a context of bound variables (arguments of a definition, let-binding, iterators). In the general case, a type judgment is a relation between:

- a type environment \(\Gamma\): a mapping between bound variables and their type. (e.g. \(I :: \text{int}\), \(L :: [\text{int}]\))
- an expression \(e\)
- the type \(t\) of \(e\) under \(\Gamma\)

A type judgement is denoted:

\[\Gamma \vdash e :: t\]

e.g. \(I :: \text{int} \vdash [I] :: [\text{int}]\)
Cream Typing Rules

Basis:

- Integer constants
  \[ \Gamma \vdash n :: int \geq 2 \quad n \in \mathbb{N} \quad \Gamma \vdash 0 :: bool \quad \Gamma \vdash 1 :: bool \]

- Bound variables, FD variables, empty lists.
  \[ X :: \tau, \Gamma \vdash X :: \tau \quad \Gamma \vdash X :: int \quad X \notin \Gamma \quad \Gamma \vdash [] :: \left[ \tau \right] \quad \tau \text{ type} \]

Inductive steps: hypotheses are on top of the line, conclusions on bottom

\[ \Gamma \vdash X_1 :: \tau \cdots \Gamma \vdash X_n :: \tau \quad \Gamma \vdash X_1 :: \tau_1 \cdots \Gamma \vdash X_n :: \tau_n \]
\[ \Gamma \vdash \{ f_1 = X_1, \ldots, f_n = X_n \} :: \{ f_1 : \tau_1, \ldots, f_n : \tau_n, uid : int \} \]
\[ \Gamma \vdash e_1 :: int \quad \Gamma \vdash e_2 :: int \quad \Gamma \vdash e :: \{ f_1 : \tau_1, \ldots, f_n : \tau_n \} \quad \Gamma \vdash e : f_i :: \tau_i \]
Type Coercions

We make coercions explicit with a new syntactic construction: $\mu$

- Reification: `bool` is a subtype of `int`

  $$
  \Gamma \vdash e :: bool \\
  \Gamma \vdash \mu_{bool \rightarrow int}(e) :: int
  $$

- Projection: $\{f: \tau\}$ is a subtype of $\{f: \tau, g: \tau'\}$

  $$
  \Gamma \vdash e :: \{f_1 : \tau_1, \ldots, f_n : \tau_n\} \\
  \Gamma \vdash \mu_{\pi}(e) :: \{f_{\pi_1} : \tau_{\pi_1}, \ldots, f_{\pi_k} : \tau_{\pi_k}\}
  $$
Typing Definitions and Calls

Typing a call should be equivalent to type the body of the definition given the arguments as environment:

\[\Gamma \vdash e_1 :: \tau_1 \cdots \Gamma \vdash e_n :: \tau_n \quad (f(X_1, \ldots, X_n) = e) \in P \quad X_1 : \tau_1, \ldots, X_n : \tau_n \vdash e :: \tau\]

\[\Gamma \vdash f(e_1, \ldots, e_n) :: \tau\]

**Goal:** associate to the definition \(f(X_1, \ldots, X_n) = e\) a principal type, that is to say a type valid for this definition that is more general than any other valid type. With subtyping, the principal type between bool and int is int. But there are definitions which can take either int, or [int], or...: \(\text{id}(X) = X\). Principality comes from the fact that if Cream Typing Rules are such that if a definition can be called with two unrelated type (e.g. int, or [int]), it can be called with any type \(\tau\).

Let \(\alpha, \beta, \ldots\) be a countable set of type variable. A **type schema** is of the form:

\[\forall \alpha \beta \ldots (f(\tau_1, \ldots, \tau_n) :: \tau)\]

Since Cream definition are top-level, all type schema are closed.

Identity definition has type: \(\forall \alpha, (id(\alpha) :: \alpha)\)
Rule declarations and object definition

- **Rule declarations** (and queries) define constraints, the type system enforces that they return \texttt{bool}.

- **Object definition** define data structure, the type system enforces that they don’t return \texttt{bool}.

Consequence: \( f = 1 \) has type \( f :: \texttt{int} \) (by coercion) whereas \( p \rightarrow 1 \) has type \( p :: \texttt{bool} \). \( p \) is usable as a predicate, \( f \) isn’t.
Type Unification

Goal: Guess the type of the arguments of a definition.

Use type variable as (yet) unknown type.

\[
X_1: \alpha_1, \ldots, X_n: \alpha_n \vdash e :: \alpha
\]

\[
\Gamma \vdash f(X_1, \ldots, X_n) = e :: f(\alpha_1, \ldots, \alpha_n) : \alpha
\]

Typing rules enforce equality between types.

Row variables: for (yet) unknown fields of a record.

\[
\Gamma \vdash e :: \{ f : \tau, \rho \}
\]

\[
\Gamma \vdash e : f :: \tau
\]

Generalization to type schema for definitions.
bool → int  Coercion in a Hindley-Milner framework

Type constructors value(bool) and value(int).

Generalization of value(bool) to value(α) in type schema.

μbool→int only introduced on predicate arguments and value-let.
let (X = (1 = 1), X and X = 1) is ill-typed: the first usage of X has type value(bool) whereas the second has type value(int).
Conclusion

- Early detection of errors
- Coding discipline:
  - Homogeneous data structures
  - Enforces separation between rule declarations and object definitions
- Type inference could be less restrictive on coercions with a Cardelli/Mitchell algorithm.